6-4 Videos Guide

6-4a

- Area in polar coordinates
  - $A = \int_a^b \frac{1}{2}[f(\theta)]^2 \, d\theta$
  - Between two curves f and g ($f \geq g$): $A = \frac{1}{2}\int_a^b \{[f(\theta)^2 - [g(\theta)]^2\} \, d\theta$

- Arc length
  - $L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \, d\theta = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$

Exercises:

6-4b

- Graph the curve and find the area that it encloses.
  $r = 3 - 2\cos 4\theta$

6-4c

- Find the area of the region that lies inside the first curve and outside the second curve.
  $r = 3\sin\theta, \quad r = 2 - \sin\theta$

6-4d

- Find the area of the region that lies inside both curves.
  $r = \sin 2\theta, \quad r = \cos 2\theta$

6-4e

- Find the length of the polar curve.
  $r = 5^\theta, \qquad 0 \leq \theta \leq 2\pi$

6-3f

- Tangents to a polar curve: if $r = f(\theta)$ and $x = r\cos\theta, \quad y = r\sin\theta$, then
  - $\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$ (by the product rule)
  - $\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$ (product rule again)
  - $\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$
  - Horizontal tangents occur when $\frac{dy}{d\theta} = 0$ (if $\frac{dy}{dx} \neq 0$)
  - Vertical tangents occur when $\frac{dx}{d\theta} = 0$ (if $\frac{dx}{dy} \neq 0$)

  Note: if both $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} = 0$, we use l'Hospital's Rule

Exercises:

- Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.
  $r = 2 + \sin 3\theta, \qquad \theta = \pi/4$

6-3g

- Find the points on the given curve where the tangent line is horizontal or vertical.
  $r = 1 - \sin\theta$